HYPERSONIC FLOW OF A NONEQUILIBRIUM-IONIZED RADIATING GAS PAST BLUNT BODIES

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Below we investigate the axisymmetrical hypersonic flow of a nonequilibrium-ionized inviscid radiating monatomic gas past a blunt body.

The occurrence of high temperatures behind the shock wave front associated with hypersonic flow past blunt bodies generates processes such as ionization and radiation. In the majority of papers devoted to radiating gas flows the medium is considered to be in equilibrium [1, 2]. Under certain conditions, however (for example, with a decrease in the freestream pressure), the size of the relaxation zone becomes comparable with the standoff distance of the shock wave, making it necessary to calculate nonequilibrium processes.

For the description of the nonequilibrium processes in the shock layer it is necessary to apply a specific kinetic model. In the present article we rely on the kinetics proposed by Clarke and Ferrari [3] for argon. We examine the reactions $A+e \rightleftharpoons A^{+}+e+e$, $A+h\nu \rightleftharpoons A^{+}+e$. In calculating the photoionization reaction we consider only bound-free radiation from the continuum region ($\nu \geq \nu_j$), neglecting radiation in the lines.

Inasmuch as the shock standoff distance is much less than the characteristic scale of the body, we use the locally one-dimensional plane layer approximation for the calculation of the radiation parameters. We analyze the influence of the freestream parameters on the flow field in the shock layer and the distribution of the radiative heat flux.

NOTATION

θ) Polar angle; $\mathbf{r}$) radius vector measured from the center of curvature of the bow of the body; $\mathbf{r}_T$, $\mathbf{r}_b$) radius vectors of the surfaces of the body and shock wave; ε) shock standoff distance, $\varepsilon(\theta) = r_b - r_T$; L) characteristic scale (radius of curvature of the body at the bow); u, v) components of the gas velocity along the radius vector and along the normal to the body; W) total-velocity modulus; W_n, W_t) normal and tangential velocity components relative to the shock wave; W_m) peak gas velocity (exit flow velocity in vacuum) p) gas pressure; ρ) gas density; H) total enthalpy per gram of mixture; α) degree of nonequilibrium ionization of the gas; α_E) degree of equilibrium ionization of the gas; T) gas absolute temperature (°K); ν) frequency; m_a) atomic mass; ν_j, T_j) ionization frequency and temperature; $\mathbf{q}_\nu$, $\mathbf{q}$) spectral and total radiative energy flux vectors:

$$\mathbf{q} = \int_0^\infty \mathbf{q}_\nu d\nu;$$

$\varkappa_\nu$) spectral coefficient of absorption per unit mass of atomic gas; $B_\nu(T)$) Planck constant; C_i, C_n) collisional ionization and recombination rate constants; δ) blackness coefficient (fourth-power-law coefficient) of the body surface; I_ν^+, I_ν^-) spectral intensity of radiation propagating in the positive (+) and negative (−) directions of the ξ axis; R) specific gas constant; ∞, 0, b) subscripts referring to the gas parameters in the unperturbed (freestream) flow, on the body, and in front of the shock wave, respectively;

$$a = \frac{\varepsilon}{r}, \quad U = u - Vv, \quad V = (r_T + \xi\varepsilon)^{-1}\left(\frac{dr_T}{d\theta} + \xi\frac{d\varepsilon}{d\theta}\right)$$

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1. INITIAL SYSTEM OF EQUATIONS

We introduce the dimensionless variables

$$\bar{u} = \frac{u}{W_m}, \quad \bar{v} = \frac{v}{W_m}, \quad \bar{\rho} = \frac{\rho}{\rho_\infty}, \quad \bar{p} = \frac{p}{\rho_\infty W_m{}^2}$$
$$\bar{q} = \frac{2q}{\rho_\infty W_m{}^3}, \quad \bar{T} = \frac{RT}{W_m{}^2}, \quad \bar{r} = \frac{r}{L}$$

Now in the coordinate system θ, ξ, where $\xi = (r - r_T)/(r_b - r_T)$, the initial system of equations describing the flow of a nonequilibrium-ionized radiative gas past a blunt body has the form (the bars over the dimensionless variables are dropped)

$$\frac{\partial u}{\partial \xi} + \frac{U}{\rho}\frac{\partial \rho}{\partial \xi} - V\frac{\partial v}{\partial \xi} + a\left(\frac{\partial v}{\partial \theta} + 2u + v\operatorname{ctg}\theta + \frac{u}{\rho}\frac{\partial \rho}{\partial \theta}\right) = 0 \tag{1.1}$$

$$U\frac{\partial u}{\partial \xi} + \frac{1}{\rho}\frac{\partial p}{\partial \xi} + a\left(v\frac{\partial u}{\partial \theta} - v^2\right) = 0 \tag{1.2}$$

$$U\frac{\partial v}{\partial \xi} - \frac{V}{\rho}\frac{\partial p}{\partial \xi} + a\left(v\frac{\partial v}{\partial \theta} + \frac{1}{\rho}\frac{\partial p}{\partial \theta} + uv\right) = 0 \tag{1.3}$$

$$\rho\frac{U}{\varepsilon}\frac{\partial H}{\partial \xi} + \rho\frac{v}{r}\frac{\partial H}{\partial \theta} = -\nabla \mathbf{q} \tag{1.4}$$

$$\rho\frac{U}{\varepsilon}\frac{\partial \alpha}{\partial \xi} + \rho\frac{v}{r}\frac{\partial \alpha}{\partial \theta} = -m_a\frac{1}{\varepsilon}\frac{\partial}{\partial \xi}\int_{\nu_j}^{\infty}(h\nu)^{-1}q_\nu d\nu + \alpha(1-\alpha)\rho^2 C_i{}^{-1}T^{1/2}\exp(-T_j/T) - \alpha^3\rho^3 C_n{}^{-1}T^{-1} \tag{1.5}$$

$$\frac{\mu}{\varepsilon}\frac{\partial I_\nu}{\partial \xi} = \rho(1-\alpha)\varkappa_\nu\left[\frac{\alpha^2}{1-\alpha}\frac{1-\alpha_E}{\alpha_E{}^2}B_\nu - I_\nu\right] \tag{1.6}$$

$$p = \rho T(1+\alpha) \tag{1.7}$$

The following boundary conditions are imposed:
on the wave:

$$W_{\infty n} = \rho W_n, \qquad W_{\infty t} = W_t$$
$$p_\infty + W_{\infty n}^2 = p + \rho W_n{}^2 \tag{1.8}$$

$$5T_\infty(1+\alpha_\infty) + 2\alpha_\infty T_j + W_\infty{}^2 + q_b/W_\infty = 5T(1+\alpha_\infty) + 2\alpha_\infty T_j + W^2 + q_b/W_\infty$$

on the body:

$$u = 0 \tag{1.9}$$

In order to obtain the final expression for the radiative energy flux it is required to formulate boundary conditions for the spectral radiation intensity. Since we are using the locally one-dimensional plane layer approximation, we need two such conditions.

The gas in front of the shock wave is only slightly heated, and it may therefore be assumed that it is nonradiating, i.e.,

$$I_{\nu b}^- = 0 \tag{1.10}$$

We use the balance of radiative energy for the boundary condition on the body

$$I_{\nu 0}^+ = \delta B_\nu(T_m) + (1-\delta)I_{\nu 0}^- \tag{1.11}$$

where T_m is the surface temperature of the body.

Bearing all of the foregoing in mind and carrying out suitable computations, we obtain the following analytical expression for the spectral radiation intensity:

$$I_\nu{}^+(\tau_\nu, \mu) = \int_0^{\tau_\nu} S_\nu \exp\left(-\frac{\tau_\nu - t}{\mu}\right)\frac{dt}{\mu} + [\delta B_\nu(T_m) + (1-\delta)I_{\nu 0}{}^-]\exp(-\tau_\nu/\mu) \tag{1.12}$$

Invoking the dependence of the radiative flux on the spectral intensity, we obtain

$$q^+(\tau_\nu) = 2\pi\left\{\int_0^{\tau_\nu}\int_0^1 \frac{\alpha^2}{1-\alpha}\,\frac{1-\alpha_E}{\alpha_E^2}\,B\exp\left(-\frac{\tau_\nu - t}{\mu}\right)d\mu\,dt\right.$$

$$\left.+\,\delta B(T_m)\int_0^1 \mu\exp\frac{-\tau_\nu}{\mu}\,d\mu + (1-\delta)\int_0^{\tau_b}\int_0^{\mu}\frac{\alpha^2}{1-\alpha}\,\frac{1-\alpha_E}{\alpha_E^2}\,B\exp\left(-\frac{t+\tau_\nu}{\mu}\right)d\mu\,dt\right\} \tag{1.13}$$

Here

$$\tau_\nu = \varepsilon\int_0^{\xi}\varkappa_\nu\rho(1-\alpha)\,d\xi \quad \text{(optical coordinate)}$$

$$S = \int_{\nu_j}^{\nu_k} S_\nu d\nu = \int_{\nu_j}^{\nu_k}\frac{\alpha^2}{1-\alpha}\,\frac{1-\alpha_E}{\alpha_E^2}\,B_\nu d\nu \quad \text{(source function)}$$

$$B = \int_{\nu_j}^{\nu_k} B_\nu d\nu = \frac{2k^4T^4}{h^3c^2}\left\{\exp\left(-\frac{h\nu_j}{kT}\right)\left[\left(\frac{h\nu_j}{kT}\right)^3 + 3\left(\frac{h\nu_j}{kT}\right)^2 + 6\,\frac{h\nu_j}{kT} + 6\right]\right.$$

$$\left. - \exp\left(-\frac{h\nu_k}{kT}\right)\left[\left(\frac{h\nu_k}{kT}\right)^3 + 3\left(\frac{h\nu_k}{kT}\right)^2 + 6\,\frac{h\nu_k}{kT} + 6\right]\right\} \tag{1.14}$$

For radiation propagating in the angular interval $(0, \frac{1}{2}\pi)$ we deduce the expression

$$I_\nu^-(\tau_\nu;\mu) = -\int_{\tau_\nu}^{\tau_{\nu b}}\frac{\alpha^2}{1-\alpha}\,\frac{1-\alpha_E}{\alpha_E^2}\,B_\nu\exp\left(-\frac{\tau_\nu - t}{\mu}\right)\frac{dt}{\mu} \tag{1.15}$$

On the basis of the definition of the total radiative flux, relation (1.15), and the substitution $\mu = -\mu$ we have

$$q^-(\tau_\nu) = 2\pi\int_{\tau_\nu}^{\tau_{\nu b}}\int_0^1\frac{\alpha^2}{1-\alpha}\,\frac{1-\alpha_E}{\alpha_E^2}\,B\exp\left(-\frac{\tau_\nu - t}{\mu}\right)d\mu\,dt \tag{1.16}$$

To determine the contribution of radiation to the energy equation and relaxation we use the relations

$$\nabla q = \frac{1}{\varepsilon}\,\frac{dq}{d\xi}, \qquad q = q^+ - q^- \tag{1.17}$$

In order to take radiation anisotropy into account we use the following dependence of the radiation spectral intensity on the direction of photon propagation: The total angular interval $(0, \frac{1}{2}\pi)$ is partitioned into N zones, in each of which μ_i is the mean cosine of the angle for the given zone, i = 1, 2, ..., N. The radiation in each zone is assumed to be isotropic.

From relations (1.13), (1.16), and (1.17), substituting summation for integration over μ, we obtain the final results

$$q(\tau_\nu) = 2\pi\sum_{i=0}^{N-1}(\mu_{i+1} - \mu_i)\left[F_1^i(\tau_\nu) + \delta Q_2^i(\tau_\nu) + (1-\delta)F_2^i(\tau_\nu) + F_3^i(\tau_\nu)\right] \tag{1.18}$$

$$\nabla q = 2\pi\rho(1-\alpha)\varkappa_\nu\sum_{i=0}^{N-1}\left(\frac{\mu_{i+1}-\mu_i}{\mu_i}\right)\left[-F_1(\tau_\nu) - \delta Q_2^i(\tau_\nu)\mu_i - (1-\delta)F_2^i(\tau_\nu) - F_3^i(\tau_\nu)\right] + 4\pi\rho(1-\alpha)\varkappa_\nu Q_1(\tau_\nu) \tag{1.19}$$

Here

$$F_1^i(\tau_\nu) = \exp\left(-\frac{\tau_\nu}{\mu_i}\right)\int_0^{\tau_\nu}\frac{\alpha^2}{1-\alpha}\,\frac{1-\alpha_E}{\alpha_E^2}\,B\exp\left(\frac{t}{\mu_i}\right)dt \tag{1.20}$$

$$F_2^i(\tau_\nu) = \exp\left(-\frac{\tau_\nu}{\mu_i}\right)\int_0^{\tau_{\nu b}}\frac{\alpha^2}{1-\alpha}\,\frac{1-\alpha_E}{\alpha_E^2}\,B\exp\left(-\frac{t}{\mu_i}\right)dt \tag{1.21}$$

$$F_3^i(\tau_\nu) = \exp\left(\frac{\tau_\nu}{\mu_i}\right)\int_0^{\tau_{\nu b}}\frac{\alpha^2}{1-\alpha}\,\frac{1-\alpha_E}{\alpha_E^2}\,B\exp\left(-\frac{t}{\mu_i}\right)dt \tag{1.22}$$

$$Q_2^i(\tau_\nu) = B(T_m)\exp\left(-\frac{\tau_\nu}{\mu_i}\right) \tag{1.23}$$

$$Q_1(\tau_\nu) = \frac{\alpha^2}{1-\alpha}\frac{1-\alpha_E}{\alpha_E^2}B \tag{1.24}$$

In view of the singularity imposed by the integral character of the radiation terms, it becomes necessary to solve the problem iteratively. The blunt-body flow problem is reduced to a double iterative procedure, because the position and shape of the shock wave are also unknown beforehand.

In connection with the fact that the ionization of the gas is of an avalanche character and the degree of ionization α of the gas has large gradients across the shock wave, it was decided to use a solution scheme in which the unknown functions are approximated along the shock wave and the integration is carried out transversely to the shock layer. The system of differential equations solved for the derivatives with respect to ξ was integrated along three ways ($\theta = 0$, 0.25, and 0.5 rad) from the wave ($\xi = 1$) to the body ($\xi = 0$) by the Euler method with multiple scaling and without subdivision of the integration step. The integration step could be chosen differently on different intervals, depending on the behavior of the functions to be integrated on the given interval.

In the stated problem only radiation transfer from the continuum zone is taken into account. Inasmuch as $\varkappa_\nu \sim \nu^{-2}$ in this zone, it is fully justified to use a step-function approximation for the absorption coefficient. In this approximation the frequency interval from $\nu = 0$ to $\nu = \infty$ is partitioned into a series of spectral intervals, in each of which the absorption coefficient is constant.

2. DISCUSSION OF THE RESULTS

In order to analyze the influence of the Mach numbers M_∞, initial pressure p_∞, radius L of a spherical body, and degree of ionization α_b in the free stream we carried out the calculations over a wide range of variation of the indicated parameters for a step-function model of the absorption coefficient $\varkappa$. The number of steps n in the calculations was varied from one to six; it turned out that the influence of n on the most sensitive factor, i.e., the radiation flux, is scarcely felt for $n \geq 3$. Therefore, the greater body of the calculations was carried out for the three-step approximation of $\varkappa(\nu)$. Most of the calculations were carried out on the assumption that the surface blackness coefficient $\delta = 0.5$.

The results of the calculations are presented in Fig. 1 in the form of distributions of the various gas-dynamical parameters as a function of ξ. Figures 1.1, 1.3, 1.4, and Fig. 2 illustrate the influence of the numbers M_∞ ($17 \leq M_\infty \leq 28$, 5.4 km/sec $\leq W_\infty \leq$ 8.6 km/sec) on the profiles of the gas-dynamical parameters for a sphere L = 1 cm at T = 290°K, $p_\infty = 0.01$ atm, and $\alpha_b = 10^{12}$; these figures reflect the behavior of the parameters along the zeroth streamline ($\theta = 0$); the variation of $\alpha(\xi)$ and $q(\xi)$ for various values of the angle θ (M_∞ = 24) is given in Figs. 1.5 and 1.2.

We begin with a discussion of the results of the flow calculation without regard for radiation (dashed curves in the figures). As indicated in Figs. 1.4 and 1.3, the flow field across the shock layer is divided into two zones:

a) the relaxation zone, which terminates in a surface of abrupt variation of the parameters (the appearance of very large gradients of the gas-dynamical parameters in the flow field is attributable to the avalanche character of the collisional ionization);

b) the equilibrium zone adjacent to the body.

With an increase in M_∞ the relaxation zone shrinks, and for $M_\infty > 28$ the flow may be regarded as equilibrium over the entire flow region from the wave to the body. The decrease in length of the relaxation zone with increasing M_∞ is explained by an increase in the values of T and ρ behind the shock wave and the concomitant decrease of the mean free paths of electrons, atoms, and ions and, hence, the more rapid ionization and recombination processes.

Allowance for radiation does not qualitatively alter the variation of the parameters across the shock layer, but a certain departure from the flow of a nonradiating gas is observed.

The length of the relaxation zone is decreased. This event is obviously attributable to the fact that the photons emitted in the high-temperature region reach the wave, and the photoionization reaction sets in, whereupon rather quickly a sufficient number of "seed" electrons ($\alpha \approx 10^{-4}$ to 10^{-3}) are created to initiate the avalanche formation of ions, resulting in almost instantaneous equilibrium.

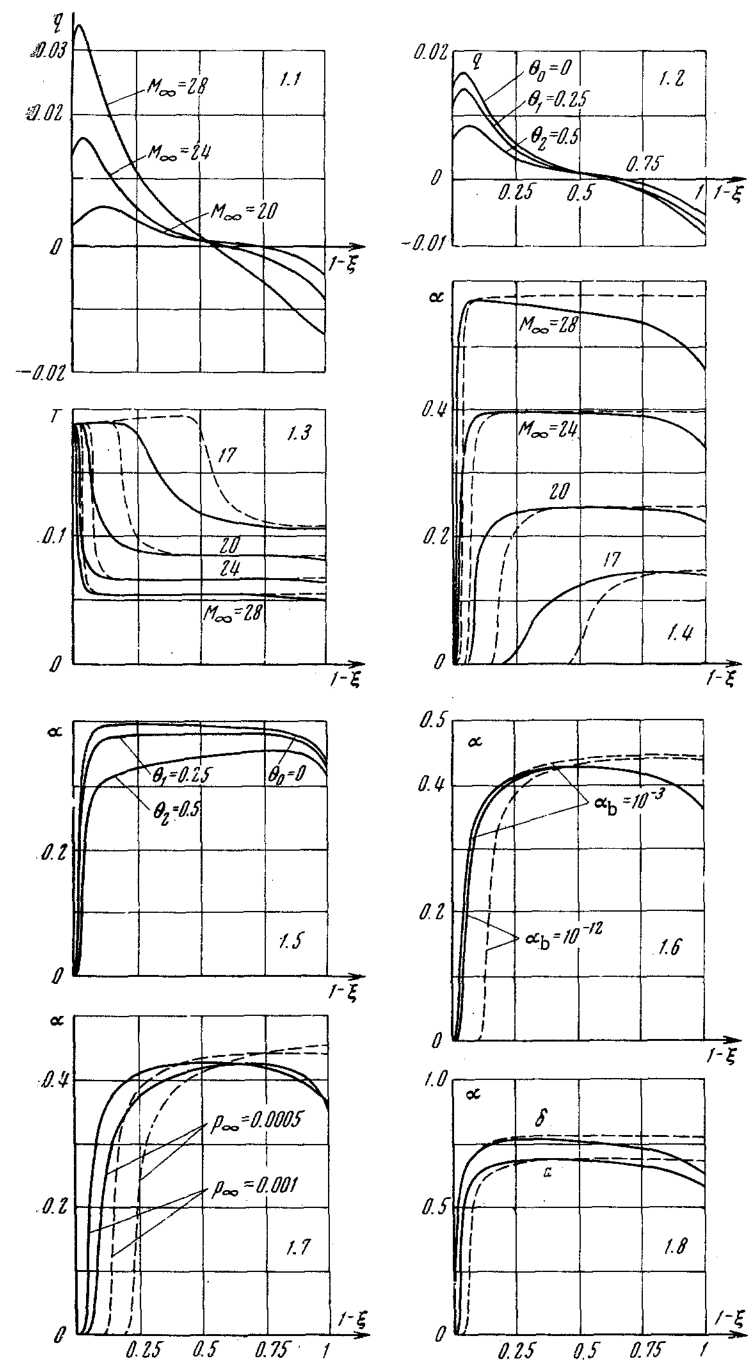

Fig. 1

With radiation taken into account a reduced-temperature layer is observed in front of the body. This layer has been calculated in [4] and has become known as the radiation entropy layer. Its onset is attributable to the withdrawal of radiative energy from the layer adjacent to the body surface.

Since the degree of ionization decreases with the temperature in the equilibrium zone, a decrease in the degree of ionization is observed in the radiation entropy layer (Fig. 1.4). In this layer a certain additional compression of the gas is observed in connection with the temperature reduction.

Profiles of the radiative fluxes on the ray $\theta = 0$ for various values of M_∞ are shown in Fig. 1.1. The q profile has a nonmonotonic character, the position of max q coinciding with the position of the avalanche-ionization front and approaching the wave surface as M_∞ is increased. It is also obvious that as it moves across the shock wave the flux changes sign; from zones adjacent to the wave the flux transports energy up-

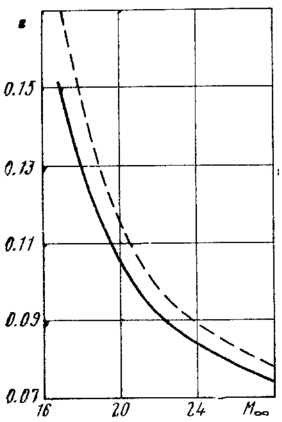

Fig. 2

stream across the shock wave; from zones adjacent to the body it transports energy into the body.

It is evident from Fig. 1.1 that as the number M_∞ is increased the flux entering the body increases in absolute value. This result is attributable to the increase in the temperature values in the compressed layer. Figure 1.2 shows the radiative flux profiles for various angular coordinates of the calculated rays. Note that the radiative fluxes diminish as they move past the body away from the symmetry axis.

The dependence of the shock standoff distance on M_∞ is given in Fig. 2. As M_∞ is increased the standoff distance falls off sharply due to the increased density behind the shock wave.

It is seen that with radiation taken into account the shock wave approaches the body. Clearly, this effect is elicited by a violation of flow adiabaticity, the transport of radiative energy from the compressed layer, and the greater rapidity of the nonequilibrium processes (as we know, for equilibrium flow the standoff distances are smaller than in nonequilibrium flow).

In order to analyze the dependence of the flow field on the freestream pressure we carried out calculations in the range $5 \cdot 10^{-4} \le p_\infty \le 10^{-3}$ atm. The results of our calculations of the degree of ionization α are given in Fig. 1.7 for the following sets of conditions: $M_\infty = 24$, $p_\infty = 5 \cdot 10^{-4}$ and 10^{-3} atm; $T_\infty = 290°K$, $\alpha_b = 10^{-12}$, and $L = 4$ cm. It is seen that as p_∞ is decreased the size of the relaxation zone increases.

In order to take the influence of the advance radiation into account we varied the degree of ionization α_b of the free stream. As an example, Fig. 1.6 shows profiles of the degree of ionization on the zeroth streamline for $M_\infty = 24$, $p_\infty = 0.001$ atm, $T_\infty = 290°K$, $L = 4$ cm, and $\alpha_b = 10^{-3}$ and 10^{-12}. It is seen that only in the absence of radiation, when the only process promoting equilibrium is the process of electron-atom collisions, is the influence of the initial degree of ionization on the width of the relaxation zone at all significant. When radiation is taken into account, a variation of α_b between the limits 10^{-12} and 10^{-3} has little effect on the equilibration rate.

For a more detailed explanation of the influence of the advance radiation on the nature of the flow in the compressed layer we analyzed a flow of argon past a sphere of radius $L = 4$ cm with freestream conditions corresponding to the calculations of Clarke and Ferrari [3] for a direct shock wave. The calculations were carried out in two variants:

a) The parameters corresponding to unperturbed flow in the variant of [3] were used to characterize the flow in front of the shock wave, viz.,

$$M_b = M_\infty = 28.9,\ W_b = W_\infty = 9.34\ \text{km/sec}p_b = p_\infty = 0.001\ \text{atm},\ T_b = T_\infty = 300°\text{K},$$
$$\alpha_b = \alpha_\infty = 10^{-12}$$

i.e., in this variant the action of the radiative flux emerging from the compressed layer on the freestream flow was completely ignored.

b) The data of the solution in [3] with regard for advance radiation were used as initial data, viz., $T_b = 1380°K$, $\alpha_b = 0.0977$, and $\rho_b = \rho_\infty$

A comparison of the calculations for variants a) and b) and the solution of Clarke and Ferrari exhibited the following.

The distribution of the gas-dynamical parameters on the zeroth streamline in variant b) is in good agreement with the solution for a direct shock wave. The maximum value of the degree of ionization of argon in both cases is equal to 77% and is attained at a distance of order 0.1 to 0.2 times the average radiation path from the shock wave. The equilibrium temperature and density values in these variants agree to within 1 or 2%.

A comparison of the solutions in cases a) and b) (Fig. 1.8) shows that with the advance radiation taken into account the profile of the degree of ionization α is 8 to 10% higher throughout the compressed layer. The fact that the electron concentration immediately behind the shock wave with advance radiation taken into account is considerably higher causes a sharp decrease in the width of the ionization relaxation zone and brings the flow regime close to equilibrium.

Consequently, in calculations of the flow of nonequilibrium-ionized radiating argon past blunt bodies, it is essential to make allowance for the advance radiation.

LITERATURE CITED

1. O. M. Belotserkovskii, L. M. Biberman, S. Ya. Bronin, A. N. Lagar'kov, and V. N. Fomin, "Heating of blunt bodies in a hypersonic gas flow with allowance for radiation transport," Teplofiz. Vys. Temp., 7, No. 3, 529-541 (1969).
2. R. Goulard, R. E. Boughner, R. C. Burns, and H. F. Nelson, "Flow of radiating gas around a probe entering a planetary atmosphere," Teplofiz. Vys. Temp., 7, No. 3, 542-565 (1969).
3. J. H. Clarke and C. Ferrari, "Gas dynamics with nonequilibrium radiative and collisional ionization," Phys. Fluids, 8, No. 12, 2121-2139 (1965).
4. V. K. Vertushkin, "Supersonic flow of air past a sphere with allowance for equilibrium radiation," Kosmich. Issled., 6, No. 1 (1966).